Name: **Math 210** KEY **Practice Test #2**

Directions: For each problem state the Population and Parameters and then conduct the appropriate hypothesis test using a 5% level of significance for each test.

1) 100 Randomly selected Americans over the age of 18 were asked how many hours a night they sleep and the 100 Americans had a mean sleep time of 6.8 hours and a standard deviation of 0.9 hours. An MIT professor believed that people in Italy over the age of 18 sleep more on average then people over the age of 18 in America. So he also asked 120 randomly selected Italian citizens over the age of 18 how many hours a night they sleep. The 120 Italians had a mean of 7.1 hours of sleep and a standard deviation of 0.6 hours. Is there enough evidence to support the professor's claim?

Pop₁: All American citizens
Pop₂: All Italian citizens
Parameters: (μ_1 & μ_2) Avg hours slept

Type of Test: Two - Sample t - test
Conditions: random sample₁ ✓ random samp₂ ✓
$10n_1 <$ Pop₁ ✓ $10n_2 <$ Pop₂ ✓
$n_1 \geq 30$ ✓ $n_2 \geq 30$ ✓

$H_0: \mu_1 = \mu_2$
$H_a: \mu_1 < \mu_2$

$n_1 = 100$ $\quad n_2 = 120$
$\bar{x}_1 = 6.8$ $\quad \bar{x}_2 = 7.1$
$s_1 = 0.9$ $\quad s_2 = 0.6$

t-value ≈ -2.85
d.f $= 166$
p-value $\approx .002$

Since the p-value is less than 5%. we reject the Ho, which means the evidence suggests that people in Italy do sleep more than people in the U.S.

2) You are the manager of the packaging process at a cereal manufacturing plant. You want to determine if the cereal filling process is working properly. The process requires no corrective action if the correct amount of cereal per box is at least 368 grams. To study this, you decide to take a random sample of 45 boxes, weigh each one, and then evaluate the difference between the sample statistic and the hypothesized population parameter by comparing the mean weight from the sample to the expected population mean of 368 grams specified by the company. The sample mean is 372.5 and the sample standard deviation is 15 grams. Is there evidence that the weight is *different* from 368 grams?

Pop: All cereal boxes processed at a manufacturing plant
Param: (μ) Avg. amount of cereal (in grams) per box
Type of Test: 1-Sample t-test (use "s" not "σ")

Conditions:
Random Samp. ✓
$10n <$ Pop. Size ✓
$n \geq 30$

$\bar{x} = 372.5$
$s = 15$
$n = 45$

$H_0: \mu = 368$
$H_a: \mu \neq 368$

t-value ≈ 2.01
d.f $= 44$ $(n-1)$
p-value $\approx .05$

Since the p-value is 5%. we can reject or fail to reject. (Assume p-value is .051) Since the p-value is greater than 5%. we fail to reject the Ho, which means there is NOT enough evidence to suggest the average weight of the cereal boxes is different than what is being stated (368g)

3) Explain what a Type I error and a Type II error would be in the context of the problem and state which one you think would be worse and why? Scenario: A company is testing a new product to see whether or not they should mass produce it, the product helps lower cholesterol levels in people.

Type I: reject Ho when true

Type II: ~~reject~~ fail to reject Ho when false

Ho: Assume medicine does not work

Ha: Medicine will work

I: Make the medicine but it does not work

II: Do not make medicine, even though it would work.

4) If you get a p-value of 0.034 which of these statements are correct?

(A) We would reject Ho at the 5% level but fail to reject at the 10% level

(B) We would reject Ho at the 5% level and accept Ho at the 1% level

$\boxed{\text{(C)}}$ We would fail to reject Ho at the 1% level but reject Ho at the 5% level

(D) We would fail to reject Ho at the 5% level but reject Ho at the 1% level

(E) We would reject Ho at both the 1% and 5% level

5) What is the deciding factor as to whether we calculate a z-score or a t-score?

S.D of population is known (Z)

S.D of sample is used (t)

6) Describe the difference between a parameter and a statistic. Then draw the symbol that shows a proportion as a parameter and a proportion as a statistic.

Parameters represent the population

Statistics represent the sample

Statistic proportion: $\hat{p}$ Parameter Proportion: p

7) If out of 200 students surveyed at a school 104 of them said they like to go to the movies, what would be the value of p-hat, and is p-hat a statistic or a parameter? Using the information above what would be the approximate value of p and what would be the standard deviation of our sampling distribution sample proportion?

$\hat{p} = \frac{104}{200} = .52$ (statistic)

$\mu_{\hat{p}} = p \approx .52$

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

$= \sqrt{\frac{(.52)(.48)}{200}}$

≈ 0.0353

8) An SRS of 240 first-year college students were asked whether they applied for admission to any other college. In fact, 75% of all first-year students applied to colleges besides the one they are attending. What is the probability that the poll will be within 2 percentage points of the true p?
(Between 73% and 77%)

$p = .75$

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.75(.25)}{240}} \approx .02795$

$z = \frac{.77 - .75}{.02795}$ $z \approx .72$ $(.7642)$

$z = \frac{.73 - .75}{.02795}$ $z \approx -.72$ $(.2358)$

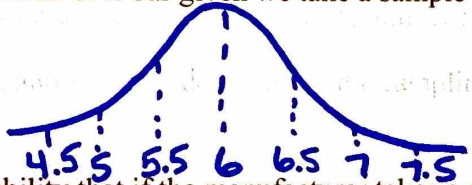
$.7642 - .2358 \approx \boxed{.5284}$

9) A population of manufactured products where the random variable X is the weight of the item. Prior experience has shown that the weight has a normal distribution with mean 6.0 ounces and standard deviation of 1.0 ounces.

a. What is the probability that the weight of "one" item randomly selected will weigh more than 7.5 ounces?

$z = \frac{7.5-6}{1} = \frac{1.5}{1} = 1.5$ $(.9332)$ $1 - .9332 = \boxed{.0668}$

b. Sketch the distribution of x-bar given we take a sample size of 4.



$\frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{4}} = \frac{1}{2} = .5$

c. What is the probability that if the manufacturer takes a sample of 100 items, that it has a mean weight between 5.85 and 6.1 ounces?

$\sigma_{\bar{x}} = \frac{1}{\sqrt{100}} = \frac{1}{10} = .1$

$.8413 - .0668 = \boxed{.7745}$

$z = \frac{5.85 - 6}{.1} = \frac{-.15}{.1} = -1.5$ $(.0668)$

$z = \frac{6.1 - 6}{.1} = \frac{.1}{.1} = 1$ $(.8413)$

10) Lie detectors are based on measuring changes in the nervous system. The assumption is that lying will be reflected in physiological changes that are not under the voluntary control of the individual. When a person is telling the truth, the galvanic skin response scores have a distribution that is normal with a mean of 51.6 and a standard deviation of 9.

- What is the probability that a sample of 10 people will have an average score less than 49.5?

$\sigma_{\bar{x}} = \frac{9}{\sqrt{10}} \approx 2.846$

Ans: $\boxed{.2296}$

$z = \frac{49.5 - 51.6}{2.846} \approx -0.74$

$(.2296)$

11) The actual time it takes to cook a 25-pound turkey is a normally distributed random variable with a **mean of 5.3** hours and a **standard deviation of 0.6 hours**.

a) What is the probability that the average cooking time of a single 25–pound turkey will take between 4.2 and 4.9 hours to cook?

$$Z = \frac{4.2-5.3}{.6} = \frac{-1.1}{.6} = -1.83 \quad (.0336) \qquad .2514 - .0336$$

$$Z = \frac{4.9-5.3}{.6} = \frac{-.4}{.6} = -0.67 \quad (.2514) \qquad \boxed{.2178}$$

b) What is the probability of the mean cooking time for a sample of 16 turkeys would be greater than 5.4 hours?

$$\sigma_{\bar{x}} = \frac{.6}{\sqrt{16}} = \frac{.6}{4} \approx 0.15 \qquad 1 - .7486$$

$$= \boxed{.2514}$$

$$Z = \frac{5.4-5.3}{0.15} \approx \frac{.1}{.15} \approx 0.67 \quad (.7486)$$

12) It is generally believed that nearsightedness affects about 14% of children. A large school district gives vision tests to 144 randomly selected incoming kindergarten children.

a) What is the mean and standard deviation of the sampling distribution?

$$\mu_{\hat{p}} = p = .14$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.14(.86)}{144}} \approx .0289$$

b) Show the conditions are met to use a normal distribution.

Random Sample ✓

$10n <$ Pop Size ✓

$(n \geq 30)$ / $np \geq 10$ & $n(1-p) \geq 10$ ✓

c) What is the probability that in this group less than 15% of the children will be found to be nearsighted?

$$Z = \frac{.15-.14}{.0289} = \frac{.01}{.0289} \approx .35 \quad (.6368)$$

13. If the probability of winning a game of chance at a carnival is 22% and you decide to play the game.

- What is the probability that you win exactly 4 out of 10 games played?

Binomial Prob

$X = 4$

$P = .22$ $n = 10$

Minitab → CALC → PROB DIST → PROB DENSITY FUNCTION

SINGLE VALUE = 4

DIST: BINOMIAL

Trials = 10 event prob = .22

$\approx \boxed{.1108}$

14. You have a bag with 10 number tiles in it and the tiles are numbered from 1 – 10. You are going to select a tile and then record the number, then put the tile back in the back and repeat the process.

- What is the probability that if you select 500 tiles you would get ***more than*** 265 odd numbered tiles?

Binomial Prob $P = .5$

$X > 265$

$n = 500$ Minitab

$1 - P(X \leq 265)$

$1 - .9172$

$\approx \boxed{.0828}$

15. What is the probability of rolling a six-sided die and getting a 4, and then getting a blue marble from a bag that contains 10 red, 7 green, 8 blue and 5 orange marbles?

$$\left(\frac{1}{6}\right)\left(\frac{8}{30}\right) = \frac{8}{180} = \frac{2}{45}$$

16. Given you have a bag that contains 100 number tiles numbered 1 – 100:

a. What would be the probability of selecting one number tile that is either even or _greater than_ 75 on one pull? (Show the probability formula you could use to solve this problem)

$$P(\text{Even}) + P(\# > 75) - P(\text{Even} \,\&\, \# > 75)$$

$$\frac{50}{100} + \frac{25}{100} - \frac{13}{100} = \frac{62}{100} = .62$$

b. Are these two events disjoint? (Explain: Why or Why Not)

No, they have some outcomes in common

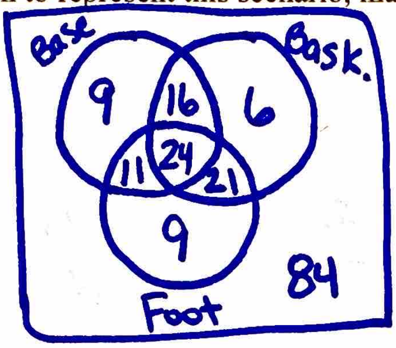
17. If you have a drawer with 16 socks in it (10 blue and 6 red), then what would be the probability of selecting two socks at random and getting a matching pair?

$$\left(\frac{10}{16}\right)\left(\frac{9}{15}\right) + \left(\frac{6}{16}\right)\left(\frac{5}{15}\right)$$

$$\frac{90}{240} + \frac{30}{240} = \frac{120}{240} = .5$$

18. At a school there are 180 students in the Senior Class and:
24 Students play Baseball, Basketball and Football
35 Students play Baseball and Football
40 Students play Baseball and Basketball
45 Students play Basketball and Football
60 Students play Baseball
65 Students play Football
67 Students play Basketball

- Draw a Venn diagram to represent this scenario, make sure to include students that <u>do not play</u> any sports.



- What is the probability that you randomly select one student and they do not play any of these sports?

$$\frac{84}{180}$$

- What is the probability of selecting one student at random that plays just Football?

$$\frac{9}{180}$$

- What is the probability that if you select two students, they both play *exactly* two sports?

$$\frac{48}{180} \cdot \frac{47}{179} \approx \boxed{.07}$$

MULTIPLE CHOICE (CIRCLE THE BEST ANSWER FOR EACH QUESTION)

19. You draw two marbles at random from a jar that has 20 red marbles and 30 black marbles without replacement. What is the probability that both marbles are red?

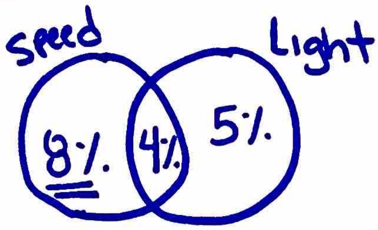
A. 0.1551
B. 0.1600
C. 0.2222
D. 0.4444
E. 0.8000

$$\left(\frac{20}{50}\right)\left(\frac{19}{49}\right) \approx .1551$$

Scenario

20. Insurance company records indicate that 12% of all teenage drivers have been ticketed for speeding and 9% for going through a red light. If 4% have been ticketed for both, what is the probability that a randomly selected teenage driver has been ticketed for speeding but not for running a red light?

A. 3%
$\boxed{\text{B.}}$ 8%
C. 12%
D. 13%
E. 17%



21. If you roll a die 150 times what is the probability that you roll between 22 and 30 fours?

$p = \frac{1}{6} = .16667$

$P(22 < x < 30)$

$n = 150$

Binomial $.8841 - .2246$

Minitab $\approx \boxed{.6595}$

$P(x \leq 30) \approx .8841$

$P(x \leq 21) \approx .2246$

22. Using the table shown for Hours worked and # of products sold, answer the following three questions.

Hours	# Sold
1	8
4	24
5	31
8	50
10	67

a) What is the equation for the linear regression line to predict products sold based on the number of hours worked?

$\hat{y} = 6.524x - .537$

b) What type of correlation does the data show?

strong positive linear correlation

c) Using your prediction equation, how many products sold would you predict for someone who worked 7 hours?

$\hat{y} = 6.524(7) - .537$

$\hat{y} \approx 45.131$