Multiplying and Dividing Rational Expressions

Warm up: Simplify the following

1) $\frac{3+6}{3} = \frac{9}{3} = 3$

2) $\frac{x^2+2x}{x} = \frac{\cancel{x}(x+2)}{\cancel{x}} = x+2$

3) $\frac{x^2-9}{x+3} = \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}} = x-3$

4) $\frac{x^2-25}{x^2+6x+5} = \frac{\cancel{(x+5)}(x-5)}{\cancel{(x+5)}(x+1)} = \frac{x-5}{x+1}$

Examples: Simplify and then state the values for x that make the expression undefined.

1) $\frac{4x^6}{2x-6} = \frac{\cancel{4}^{2}x^6}{\cancel{2}(x-3)} = \frac{2x^6}{x-3}$

$x \neq$ 3

2) $\frac{x+4}{3x^2+11x-4} = \frac{\cancel{x+4}^{1}}{(3x-1)\cancel{(x+4)}} = \frac{1}{3x-1}$

$x \neq$ $\frac{1}{3}$ or -4

3) $\frac{6x^2+7x-3}{-3x^2+x} = \frac{\cancel{(3x-1)}(2x+3)}{-x\cancel{(3x-1)}} = \frac{2x+3}{-x}$

$x \neq$ 0 or $1/3$

Examples: Multiply. Assume that all expressions are defined.

1) $\frac{x-2}{2x-3} \cdot \frac{4x-6}{x^2-4} = \frac{\cancel{x-2}}{\cancel{2x-3}} \cdot \frac{2\cancel{(2x-3)}}{(x+2)\cancel{(x-2)}} = \frac{2}{x+2}$

2) $\frac{x^2-16}{x^2-4x+4} \cdot \frac{x-2}{x^2+6x+8} = \frac{\cancel{(x+4)}(x-4)\cancel{(x-2)}}{(x-2)\cancel{(x-2)}(x+2)\cancel{(x+4)}} = \frac{x-4}{(x-2)(x+2)}$

3) $\frac{3x}{x-3} \cdot \frac{x^2-9}{12x^3} = \frac{\cancel{3}^{1}x(x+3)\cancel{(x-3)}}{\cancel{(x-3)}\,\cancel{12}_{4}x^{\cancel{3}\,2}} = \frac{x+3}{4x^2}$

4) $\frac{x^2+4x+3}{x^2-4} \cdot \frac{x^2-6x+8}{x^2-6x+8}$

$$\frac{(x+3)(x+1)\cancel{(x-4)}\cancel{(x-2)}}{(x+2)\cancel{(x-2)}\cancel{(x-4)}(x-2)} = \frac{(x+3)(x+1)}{(x+2)(x-2)}$$

Examples: Divide. Assume that all expressions are defined.

1) $\frac{x+3}{x^2-2x+1} \div \frac{x+3}{x-1}$

$= \frac{\cancel{x+3}}{(x-1)\cancel{(x-1)}} \cdot \frac{1\ \cancel{x-1}}{\cancel{x+3}}$

$= \frac{1}{x-1}$

2) $\frac{x^3}{5y} \div \frac{x}{15y^4}$

$\frac{x^{\cancel{3}\,2}}{1\,\cancel{5}y} \cdot \frac{\overset{3}{\cancel{15}}y^{\cancel{4}\,3}}{\cancel{x}}$

$= x^2 3y^3$

3) $\frac{x^2+2x+1}{x^2-3x-18} \div \frac{x^2-1}{x^2-7x+6}$

$= \frac{(x+1)\cancel{(x+1)}}{\cancel{(x-6)}(x+3)} \cdot \frac{\cancel{(x-6)}\cancel{(x-1)}}{\cancel{(x-1)}\cancel{(x+1)}}$

$= \frac{x+1}{x+3}$

Mixed Practice:

1) $\frac{4x-8}{x^2-2x} = \frac{4\cancel{(x-2)}}{x\cancel{(x-2)}}$

$= \frac{4}{x}$

2) $\frac{x+2}{x-4} \div \frac{1}{3x-12}$

$\frac{x+2}{\cancel{x-4}} \cdot \frac{3\cancel{(x-4)}}{1}$

$= 3(x+2)$

3) $\frac{x^2-2x-8}{9x^2-16} \cdot \frac{3x^2+10x+8}{x^2-16}$

$\frac{\cancel{(x-4)}(x+2)\cancel{(3x+4)}(x+2)}{(3x-4)\cancel{(3x+4)}(x+4)\cancel{(x-4)}}$

$\frac{(x+2)(x+2)}{(3x-4)(x+4)}$

4) $\frac{6x^3y^2}{7z^4} \div \frac{2xy^2}{21z^2}$

$\frac{\overset{3}{\cancel{6}}x^{\cancel{3}\,2}\cancel{y^2}}{1\,\cancel{7}z^{\cancel{4}\,2}} \cdot \frac{\overset{3}{\cancel{21}}\cancel{z^2}}{\underset{1}{\cancel{2}}\cancel{x}\cancel{y^2}}$

$= \frac{9x^2}{z^2}$

5) $\frac{x^2-36}{x^2-12x+36}$

$\frac{(x+6)\cancel{(x-6)}}{(x-6)\cancel{(x-6)}}$

$= \frac{x+6}{x-6}$

6) $\frac{4x^2-3x}{4x^2-1} \cdot \frac{2x+1}{x}$

$\frac{\cancel{x}(4x-3)\cancel{(2x+1)}}{\cancel{(2x+1)}(2x-1)\cancel{(x)}}$

$= \frac{4x-3}{2x-1}$

Adding and Subtracting Rational Expressions

Simplify and also, state x-values that make the expressions undefined.

$x \neq -3$ or 4

1) $\frac{5}{x-4}+\frac{2x}{x+3} = \frac{5(x+3)}{(x-4)(x+3)} + \frac{2(x)(x-4)}{(x-4)(x+3)}$

$$\frac{5x+15+2x^2-8x}{(x-4)(x+3)} = \frac{2x^2-3x+15}{(x-4)(x+3)}$$

2) $\frac{7x}{x^2-5x}+\frac{x^2}{x-5} = \frac{7x}{x(x-5)} + \frac{x^2(x)}{x(x-5)} = \frac{7x+x^3}{x(x-5)} = \frac{\cancel{x}(x^2+7)}{\cancel{x}(x-5)}$

$$= \frac{x^2+7}{x-5}$$

3) $\frac{2}{x+4}+\frac{x}{x-3} = \frac{2(x-3)}{(x-3)(x+4)} + \frac{x(x+4)}{(x-3)(x+4)} = \frac{2x-6+x^2+4x}{(x-3)(x+4)}$

$$= \frac{x^2+6x^2-6}{(x-3)(x+4)}$$

4) $\frac{b-3}{b^2-9}+\frac{b+3}{b^2+6b+9} = \frac{\cancel{(b-3)}1}{\cancel{(b-3)}(b+3)} + \frac{\cancel{(b+3)}1}{(b+3)\cancel{(b+3)}} = \frac{1}{b+3} + \frac{1}{b+3}$

$$= \frac{2}{b+3}$$

1) $\frac{3x+2}{x-1} - \frac{x+5}{x-1} = \frac{2x-3}{x-1}$

2) $\frac{4x-3}{x^2-9} - \frac{2x-3}{x-3} = \frac{4x-3}{(x+3)(x-3)} - \frac{2x-3(x+3)}{(x-3)(x+3)} = \frac{4x-3-(2x^2+6x-3x-9)}{(x-3)(x+3)}$

$= \frac{-2x^2+x+6}{(x-3)(x+3)}$

$= \frac{-(2x^2-x-6)}{(x-3)(x+3)}$

$= \frac{-(2x+3)(x-2)}{(x-3)(x+3)}$

3) $\frac{y-5}{3y+9} - \frac{y+1}{2y+6} = \frac{2(y-5)}{2\cdot 3(y+3)} - \frac{3(y+1)}{3\cdot 2(y+3)} =$

$= \frac{2y-10-3y-3}{6(y+3)} = \frac{-y-13}{6(y+3)}$

4) $\frac{x+1}{x^2-7x+10} + \frac{3}{x^2-x-2}$

$\frac{x+1(x+1)}{(x+1)(x-5)(x-2)} + \frac{3(x-5)}{(x-2)(x+1)(x-5)}$

$\frac{x^2+2x+1+3x-15}{(x+1)(x-5)(x-2)} = \frac{x^2+5x-14}{(x+1)(x-5)(x-2)} = \frac{(x+7)\cancel{(x-2)}}{(x+1)(x-5)\cancel{(x-2)}}$

$= \frac{x+7}{(x+1)(x-5)}$

Complex Rational Expressions

Warm up: Simplify the following 4 problems. Be sure to state restrictions on the variables.

1. $\frac{2x^2 + 11x + 5}{3x^2 + 17x + 10} = \frac{(2x+1)\cancel{(x+5)}}{(3x+2)\cancel{(x+5)}}$

$= \frac{2x+1}{3x+2}$

$x \neq \frac{-2}{3}, -5$

2. $\frac{7x - 28}{x^2 - 16} = \frac{7\cancel{(x-4)}}{(x+4)\cancel{(x-4)}}$ $\quad x \neq 4, -4$

$= \frac{7}{x+4}$

Multiply or divide. Write your answer in simplest form. Be sure to state restrictions on the variables.

3. $\frac{5a}{5a + 5} \cdot \frac{10a + 10}{a}$

$\frac{\cancel{5a} \cdot 10\cancel{(a+1)}}{\cancel{5}\cancel{(a+1)}\cancel{a}} = 10$

4. $\frac{x^2-25}{x+3} \div \frac{x+5}{x^2-9}$ $\quad \frac{\cancel{(x+5)}(x-5)}{\cancel{x+3}} \cdot \frac{\cancel{(x+3)}(x-3)}{\cancel{x+5}}$

$= (x-5)(x-3)$

Solving Rational Equations

1) $\frac{1}{x-5} = \frac{7}{2x}$

$2x = 7x - 35$

$35 = 5x$

$\boxed{7 = x}$

2) $\frac{1}{8}\frac{x}{x} + \frac{2}{x}\frac{8}{8} = \frac{17}{8x}$ $= x + 16 = 17$

$x = 1$

$\frac{1x}{8x} + \frac{2 \cdot 8}{8x} = \frac{17}{8x}$

3) $x - \frac{8}{x} = -2$

$\frac{x^2}{x} - \frac{8}{x} = -\frac{2x}{x}$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4 \quad x = 2$

4) $\frac{5}{x-3} - \frac{7}{x+4} = \frac{18}{x^2+x-12}$

$\frac{(x+4)5}{(x-3)(x+4)} - \frac{7(x-3)}{(x+4)(x-3)} = \frac{18}{(x+4)(x-3)}$

$5x + 20 - 7x + 21 = 18$

$-2x + 41 = 18$

$-2x = -23$

$x = 23/2$

5) $\frac{k}{k+5}+\frac{k}{k-2}=2$

$$\frac{K(K-2)}{(K+5)(K-2)}+\frac{K(K+5)}{(K-2)(K+5)}=\frac{2(K-2)(K+5)}{(K-2)(K+5)}$$

$$K^2-2K+K^2+5K=2(K^2+3K-10)$$

$$\cancel{2K^2}+3K=\cancel{2K^2}+6K-20$$

$$\frac{-\cancel{3}K}{-\cancel{3}}=\frac{-20}{-3} \qquad K=\frac{20}{3}$$

6) $\frac{12}{x-1}-\frac{8}{x}=2$

$$\frac{12(x)}{(x-1)(x)}-\frac{8(x-1)}{x(x-1)}=\frac{2(x)(x-1)}{1(x)(x-1)}$$

$$12x-\overset{4x}{8x}+8=2x^2-2x$$

$$\frac{0}{2}=\frac{2x^2-6x-8}{2}$$

$$0=x^2-3x-4$$

$$0=(x-4)(x+1)$$

$$\boxed{x=4 \qquad x=-1}$$

7) $\frac{6}{x^2+5x-66}-\frac{3x}{x+11}=-3$

$$\frac{6}{(x+11)(x-6)}-\frac{3x(x-6)}{(x+11)(x-6)}=\frac{-3(x+11)(x-6)}{(x+11)(x-6)}$$

$$6-3x^2+18x=-3(x^2+5x-66)$$

$$6-\cancel{3x^2}+18x=-\cancel{3x^2}-15x+198$$

$$33x=192$$

$$x=\frac{192}{33}=\frac{64}{11}$$

MAX and Min Quadratic Problems

1. After t seconds, a ball tossed in the air from the ground level reaches a height of H feet given by the equation: $\mathbf{H(t) = 144t - 16t^2}$.

   a. What is the height of the ball after 3 seconds?

   $H(3) = 144(3) - 16(3^2)$
   $= 288$ feet

   b. What is H(4)?

   $H(4) = 144(4) - 16(4^2)$
   $= 320$ feet

   c. What is the maximum height the ball will reach?

   $X = \frac{-b}{2a} = \frac{-144}{2(-16)} = \frac{-144}{-32} = \frac{9}{2} = 4.5$ sec.

   $H(4.5) = 144(4.5) - 16(4.5^2) = 324$ ft.

2. A rocket carrying fireworks is launched from a hill 80 feet above a lake. The rocket will fall into the lake after exploding at its maximum height. The rocket's height above the surface of the lake is given by $\mathbf{h = -16t^2 + 64t + 80}$.

   a. What is the height of the rocket after 1.5 seconds?

   $h = 140$

   b. How long will it take the rocket to reach the maximum height?

   $X = \frac{-b}{2a} = \frac{-64}{2(-16)} = 2$ seconds

   c. What is the maximum height reached by the rocket?

   $h = 144$ feet